Scott Christian UNIVERSITY

A Dynamic Centre of Academic Excellence

SCHOOL OF EDUCATION

BACHELOR OF EDUCATION

UNIT CODE/TITLE MAT 313: COMPLEX ANALYSIS

SEPTEMBER/DECEMBER 2022/2023 END OF SEMESTER 1 EXAMINATION

TIME: 2 HRS

INSTRUCTIONS: Answer QUESTION ONE and ANY OTHER TWO QUESTIONS

QUESTION 1(Compulsory)

a) Evaluate the following;

i) $(18+12i)+(27+7i)$ (2marks)

ii) $(-5+7i)-(11-i)$ (2marks)

iii) $(14+3i)(9+2i)$ (3marks)

iv) $\frac{23+2i}{6+5i}$ (5marks)

b) For each of the following functions of z determine whether the Cauchy Riemann equations hold;

i) $f(z) = \cos x - i \sinh y$ (5marks)

ii) $f(z) = e^x \cos y + ie^x \sin y$ (5marks)

iii) $f(z) = 2x^2 + y + i(y^2 - x)$ (3marks)

iv) $f(z) = y - 2xy + i(-x + x^2 - y^2) + x^2 - y^2 + 2ixy$ (5marks)

QUESTION 2

a) Obtain the complex integral $\int_c zdz$ where c is the straight line path from $z = 12 + 4i$ to $6 + 4i$ (5marks)

b) Solve $\oint_c \ln z dz$ where c is the unit circle (8marks)

c) Evaluate $\frac{16 + i}{24i}$ (2marks)

QUESTION 3

a) A point $m = 5 + i$ is transformed by $w = f(z) = 12z^3 + 9$; determine the new position of this point and represent both P and its image on the respective planes (7marks)

b) State and prove the Residue Theorem (8marks)

QUESTION 4

a) Consider the function $f(z) = (z - 4)\sin\frac{1}{z + 2}$

i)State the singularity of $f(z)$ (1mark)

ii) What is the kind of singularity (2marks)

iii) Expand $f(z)$ in Laurent series (10marks)

iv) State the region of convergence of the series (2marks)

QUESTION 5

For each of the following functions determine the pole and the Residues at the poles

a) $\frac{2z+1}{z^2-z-2}$ (5marks)

b) $\frac{(z+1)^2}{(z-1)^2}$ (5marks)

c) $\frac{1}{z^4+1}$ (5marks)