Scott Christian UNIVERSITY

A Dynamic Centre of Academic Excellence

SCHOOL OF EDUCATION (SEPTEMBER/DECEMBER-REGULAR MODE)

END OF SEMESTER 1(2022/2023) EXAMINATION

MAT 211 :LINEAR ALGEBRA. **TIME: 2 HRS**

INSTRUCTIONS: Answer QUESTION ONE and any TWO other questions

QUESTION 1

a) Solve the following linear systems using row-reduction method

i) $\begin{aligned} 8x+2y&=4 \\ -x-16y&=5 \end{aligned}$ (4mks)

ii) $\begin{aligned} 3x-2y&=4 \\ 6x+5y&=18 \end{aligned}$ (4mks)

b) Find the inverse of the following matrix using $(A|I)$

$$\begin{pmatrix} 1 & 6 & 3 \\ 3 & 3 & 5 \\ 3 & 9 & 12 \end{pmatrix}$$ (7mks)

c) Given that; $\hat{a}=\begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$, $\hat{b}=\begin{pmatrix} -1 \\ 8 \\ -19 \end{pmatrix}$ and $\hat{c}=\begin{pmatrix} -5 \\ -17 \\ 5 \end{pmatrix}$

i) Find the dot product of $\hat{a}\&\hat{b}$ (2mks)

ii) The cross product of the three vectors (3mks)

d) (i)What is the span of a set S of vectors (1mk)

(ii)Is $\hat{v} = \begin{pmatrix} 15 \\ 12 \\ -8 \end{pmatrix}$ in Span (S) where;

$$S = \left\{ \begin{bmatrix} 6 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -10 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 9 \\ 4 \end{bmatrix} \right\}$$

(9mks)

QUESTION 2

a) When do we say that a set of vectors is linearly dependent (1mk)

b) Test the linear dependence of the following systems;

i) $\hat{x_1} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \quad \hat{x_2} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \quad \hat{x_3} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ 2 \end{pmatrix}, \quad \hat{x_4} = \begin{pmatrix} 3 \\ 5 \\ 5 \\ 7 \end{pmatrix}$ (4mks)

ii) $\hat{x} = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}, \quad \hat{y} = \begin{pmatrix} 2 \\ 5 \\ 9 \end{pmatrix}, \quad \hat{z} = \begin{pmatrix} -3 \\ 9 \\ 3 \end{pmatrix}$ (3mks)

iii) $x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, y = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, z = \begin{bmatrix} 0 \\ 4 \\ 8 \end{bmatrix}$ (3mks)

iv) $(\cos x, \sin x, x)$ (4mks)

QUESTION 3

Using the method of determinants find the inverse of the following matrices

a) $\begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & -1 \\ -3 & 4 & -2 \end{pmatrix}$ (7mks)

b) $\begin{pmatrix} 1 & 2 & 1 \\ -2 & 1 & 8 \\ 1 & -2 & 7 \end{pmatrix}$ (8mks)

QUESTION 4

Solve the following systems using Cramers rule

a) $\begin{aligned} x+2y-z &= -15 \\ 2x-y-5z &= -4 \\ -6x+y+z &= 18 \end{aligned}$ (10mks)

b) $\begin{aligned} 4x+10y &= 22 \\ -2x+3y &= -14 \end{aligned}$ (5mks)

QUESTION 5

For each of the following vectors, determine whether w is in the span of set s, if so, write it as a linear combination of the vectors in s

a) $w = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix}$, $s = \left\{ \begin{pmatrix} 3 \\ -1 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 2 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ -11 \\ -1 \end{pmatrix} \right\}$ (7mks)

b) $w = \begin{pmatrix} -5 \\ 23 \\ 12 \\ 8 \end{pmatrix}$, $s = \left\{ \begin{pmatrix} 1 \\ -4 \\ -3 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \\ -4 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 4 \\ 3 \end{pmatrix}, \begin{pmatrix} -4 \\ 2 \\ 5 \\ 8 \end{pmatrix} \right\}$ (8mks)