Scott Christian UNIVERSITY
A Dynamic Centre of Academic Excellence

SCHOOL OF EDUCATION

BACHELOR OF EDUCATION ARTS

UNIT CODE/NAME: MAT 413: DIFFERENTIAL GEOMETRY

September-December 2020/2021 End Semester Exam -Time: 2 Hours

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Instructions: Answer Question ONE (Compulsory) and any other TWO questions.

QUESTION ONE

a) Give the definition of triple scalar product then find the volume of the parallel epiped having adjacent edges $¿\langle 1,3,1\rangle, v=\langle 0,5,5\rangle, w=\langle 4,0,4\rangle$. **(4 Marks)**

b) Find the domain of the vector valued function $(t)=\sqrt{4-t^2}i+t^2 j-6k$. **(3 Marks)**

c) Evaluate the limit $\lim_{t\to 0} e^t i+\frac{sint}{t}j+e^{-t}k$ **.** **(3 Marks)**

d) Find the intervals on which the epicycloid C given by $r(t)=(5cost-\cos 5t)i+(5sint-\sin 5t)j, 0\leq t\leq 2\pi$ is smooth and sketch it . **(4 Marks)**

e) Find the equation of the line to the curve generated by $y(t)=ti+t^2 j+t^3 j$ at $t=1$. **(4 Marks)**

f) Let $f(t)=(1+t^3)i+(2t-t^2)j+tk, g(t)=(1+t^2)i+t^3 k$, Find:

i) $f(a)\cdot f(b)$ **(4 Marks)**

ii) $f(a)\times f(b)$ **(4 Marks)**

g) Evaluate the definite integral:

$$\int_0^1 (\sqrt[3]{t}¿i+\frac{1}{t+1}j+e^{-t}k)dt¿$$ **(4 Marks)**

QUESTION TWO

a) Find the velocity vector , speed, and the acceleration vector of a particle that moves along the plane curve C described by:

$r(t)=2\cos\frac{t}{2}i+2\cos\frac{t}{2}j.$ **(3 Marks)**

b) Find the tangential and normal components of acceleration for the position function given by : $r(t)=3ti-tj+t^2k$ **(5 Marks)**

c) Find the curvature K of the curve given by: $r(t)=2ti+t^2j-\frac{1}{3}t^3j$ **(7 Marks)**

QUESTION THREE

a) Let $f(t)=(\sin t)i+tk, g(t)=(1+t^2)i+e^tj$, Find:

i) $\lim_{t\to 0}(f(t)\cdot g(t))$. **(3 Marks)**

ii) $\lim_{t\to 0}(f(t)\times g(t))$. **(3 Marks)**

b) Evaluate $\lim\ [¿t\to 2(1+3t^2)i-t^3j+k]$.

(3Marks)

c) Find the equation of the tangent and normal to the curve

$y(t)=(1+t)i-t^2j+(1+t^3)k$ at $t=1.$ **(6 Marks)**

QUESTION FOUR

a) If $u(t)=(3t^2+1)i+\sin t\,j$ and $v(t)=(\cos t)i+e^tk$, Find

i) $\frac{d}{dt}(u\cdot v)$. **(3 Marks)**

ii) $\frac{d}{dt}(u\times v)$. **(4 Marks)**

b) Let $u(t)=a¿$. Determine,

i) $\|u\|$ **(2 Marks)**

ii) $\frac{d^2u}{dt^2}$ **(3 Marks)**

iii) $\left|\frac{d^2u}{dt^2}\right|$ **(3 Marks)**

QUESTION FIVE

a) Evaluate $\int_{-1}^{1}(t¿i+t^3j+e^{-4t}k)dt¿$. **(4 Marks)**

b) Find the antiderivative of $r'(t)=\cos 2t\,i-2\sin t\,j+\frac{1}{1+t^2}k$ that satisfies the initial condition $r(0)=3i-2j+k$. **(5 Marks)**

c) Given the curve $y(t)=(3t-t^3)i+3t^2j+¿$ find its curvature and the torsion. **(6 Marks)**