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A Dynamic Centre of Academic Excellence

SCHOOL OF EDUCATION

BACHELOR OF EDUCATION ARTS

UNIT CODE/NAME: MAT 410: PARTIAL DIFFERENTIAL EQUATIONS I

September - December 2020/2021 - End Semester Exam - Time: 2 Hours

Instructions: Answer Question ONE (Compulsory) and any other TWO questions

QUESTION ONE

- a) Verify that $T(x,t) = e^{-kt} \sin x$ is a solution of the parabolic heat equation:

$$\frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad (4 \text{ Marks})$$

- b) Classify and sketch the surface given by $x - y^2 - 4z^2$. (4 Marks)

- c) Solve $\frac{y^2 z}{x} \frac{\partial z}{\partial x} + xz \frac{\partial z}{\partial y} = y^2$. (6 Marks)

- d) Describe the orthogonal trajectories of the family of curves given by $y = \frac{c}{x}$ for $c \neq 0$.
Sketch several members of each family. (5 Marks)

- e) Eliminate the arbitrary function f from the equation

$$f(x^2 + y^2 + z^2, z^2 - 2xy) \quad (6 \text{ Marks})$$

- f) Solve : $2yzdx + zxdy + xy(1+z)dz = 0$. (5 Marks)

QUESTION TWO

- a) Verify the equation $(x+z)dx + z^2 dy + (y^2 - x)dz = 0$ for integrability and solve it. **(5 Marks)**
- b) Find the Orthogonal trajectories of the family of curves given by $2x^2 - y^2 = c$ then sketch several members of each family. **(5 Marks)**
- c) Identify and sketch the quadric surface:
 $4x^2 + y^2 - 4z^2 - 16x - 6y - 16z + 9 = 0$ **(5 Marks)**

QUESTION THREE

- a) Solve $(x^2 - y^2 - z^2) \frac{dz}{dx} + 2xy \frac{dz}{dy} = 2xz$. **(10 Marks)**
- b) Find the integral curves of the equations:

$$\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2} \quad \text{(5 Marks)}$$

QUESTION FOUR

- a) Show whether or not the set of equations represents a surface and if so, find the constraint equation:
 $x = u + v$
 $y = u - v$
 $z = 4uv$ **(10 Marks)**
- b) Describe and sketch the surface: $z = \sin y$. **(5 Marks)**

QUESTION FIVE

- a) Show that the charpit's equation of the differential equation $(q^2 + 1)z^2 = 2pxz + x^2$ has an integral $qz = ax$ and find the corresponding complete integral of the equation. **(10 Marks)**
- b) Find the equation of the tangent line to the curve
 $3x^2 + y^2z + 2 = 0$
 $2xz - x^2y - 3 = 0$
at the point $P(1, -1, 1)$. **(5 Marks)**